METHOD AND APPARATUS FOR EXACT CONE BEAM COMPUTED TOMOGRAPHY

The invention relates to a computed tomography method in which an examination zone is irradiated along a helical trajectory by a conical X-ray beam. Moreover, the invention relates to a computer tomograph as well as to a computer program for controlling the computer tomograph.

5 In known methods of the kind set forth, utilizing approximations, the variation in space of the absorption or the attenuation of the radiation in the examination zone can be reconstructed from the measuring values acquired by a detector unit. Such approximations, however, give rise to artifacts in the reconstructed images, said artifacts being more pronounced as the angle of aperture of the radiation beam is larger in the direction of the axis
10 of rotation ("Artifact Analysis of Approximative Cone-Beam CT Algorithms, Medical Physics, Vol. 29, pp. 51-64, 2002).

Known exact methods are usually based on Radon inversion. They require a large amount of calculation work and give rise to discretization errors in the reconstructed images.

15 Moreover, an exact method which utilizes filtered back projection is known from "Analysis of an Exact Inversion Algorithm for Spiral Cone-Beam CT", Physics Medicine and Biology, Vol. 47, pp. 2583-2597 (E1). This method again requires a large amount of calculation work, thus giving rise to long reconstruction times.

Therefore, it is an object of the present invention to provide a method which
20 enables faster, exact reconstruction of the absorption distribution in the examination zone.

In respect of the method this object is achieved by means of a computed tomography method which comprises the steps of:

-   generating, using a radiation source, a conical radiation beam which traverses an examination zone or an object present therein,
25 -   generating a relative motion between the radiation source on the one side and the examination zone or the object on the other side, which relative motion comprises a rotation about an axis of rotation and a displacement parallel to the axis of rotation and is shaped as a helix,

-        acquiring measuring values which are dependent on the intensity in the radiation beam on the other side of the examination zone by means of a detector unit during the relative motions,

-        reconstructing a CT image of the examination zone from the measuring
5   values, in which reconstruction an exact 3D back projection comprising the following steps is carried out:

-        determining the partial derivative of measuring values of parallel rays with different radiation source positions in conformity with the angular position of the radiation source,

10 -   weighted integration of the derived measuring values along K lines,

-        multiplying all measuring values by a weighting factor which corresponds to the cosine of the cone angle of the beam associated with the relevant measuring value,

-        multiplying all measuring values by a weighting factor which corresponds to the reciprocal value of the cosine of the fan angle of the beam associated with the relevant
15 measuring value,

-        reconstructing the absorption of each object point by back projection of the measuring values.

In conformity with the method which is known from E1, prior to the back projection the measuring values must be multiplied by weighting factors which are dependent
20 on the location of the object point to be reconstructed in the examination zone. In contrast therewith, in accordance with the invention the measuring values are multiplied by weighting factors prior to the back projection, said weighting factors being dependent on the location of the measuring value on the detector unit. Because the number of object points to be reconstructed generally is much smaller than the number of detector elements, exact
25 reconstruction is thus possible while requiring a comparatively small amount of calculation work only. Moreover, as opposed to the method which is known from E1, the integration interval in the back projection in accordance with the invention is not dependent on the object point, so that it is not necessary to determine an integration interval for each object point during the back projection, thus leading to a further reduction of the amount of calculation
30 work required.

Claim 2 describes a preferred reconstruction method which involves an amount of calculation work which is smaller in comparison with other methods and which leads to a favorable image quality.

Claim 3 discloses a preferred version of the integration via a K line, notably the multiplication by means of a weighting factor which leads to a high image quality.

A computer tomograph for carrying out the method is disclosed in claim 4. Claim 5 defines a computer program for controlling a computer tomograph as claimed in claim 4.

The invention will be described in detail hereinafter with reference to the drawings. Therein

Fig. 1 shows a computer tomograph which is suitable for carrying out the method in accordance with the invention,

Fig. 2 shows a flow chart of the method in accordance with the invention,

Fig. 3 shows a PI line and a path of integration for a point in the examination zone,

Fig. 4 shows the PI line and the path of integration for a point in the examination zone projected in a plane perpendicular to the axis of rotation,

Fig. 5 shows parallel beams with different beam positions,

Fig. 6 shows a K plane and a K line,

Fig. 7 is a diagrammatic view of two different radiation source positions of the radiation source, a point in the examination zone and the trajectory projected in a plane perpendicular to the axis of rotation, and

Fig. 8 shows the fan beams formed by rebinning in parallel planes.

The computer tomograph shown in Fig. 1 comprises a gantry 1 which is capable of rotation about an axis of rotation 14 which extends parallel to the z direction of the co-ordinate system shown in Fig. 1. To this end, the gantry 1 is driven by a motor 2 at a preferably constant, but adjustable angular speed. A radiation source S, for example, an X-ray source, is mounted on the gantry 1. The radiation source is provided with a collimator arrangement 3 which forms a conical radiation beam 4 from the radiation generated by the radiation source S, that is, a radiation beam which has a finite dimension other than zero in the z direction as well as in a direction perpendicular thereto (that is, in a plane perpendicular to the axis of rotation).

The radiation beam 4 traverses a cylindrical examination zone 13 in which an object, for example, a patient on a patient table (both not shown) or also a technical object may be situated. After having traversed the examination zone 13, the radiation beam 4 is incident on a detector unit 16 which is attached to the gantry 1 and comprises a detector

5   surface which comprises a plurality of detector elements arranged in rows and columns in the form of a matrix in the present embodiment. The detector columns extend parallel to the axis of rotation 14. The detector rows are situated in planes which extend perpendicularly to the axis of rotation, that is, preferably on an arc of a circle around the radiation source S. However, they may also have a different shape, for example, an arc of a circle around the

10   axis of rotation 14 or be rectilinear. Generally speaking, the detector rows comprise more detector elements (for example, 1000) than the detector columns (for example, 16). Each detector element struck by the radiation beam 4 produces a measuring value for a ray of the radiation beam 4 in each position of the radiation source.

The angle of aperture of the radiation beam 4, denoted by the reference α_{max},

15   determines the diameter of the object cylinder within which the object to be examined is situated during the acquisition of the measuring values. The angle of aperture is defined as the angle enclosed by a ray situated at the edge of the radiation beam 4 in a plane perpendicular to the axis of rotation with respect to a plane defined by the radiation source S and the axis of rotation 14. The examination zone 13, or the object or the patient table, can be

20   displaced parallel to the axis of rotation 14 or to the z axis by means of a motor 5. As an equivalent, however, the gantry could also be displaced in this direction.

When a technical object is concerned instead of a patient, the object can be rotated during an examination while the radiation source S and the detector unit 16 remain stationary.

25   When the motors 2 and 5 operate simultaneously, the radiation source S and the detector unit 16 describe a helical trajectory relative to the examination zone 13. However, when the motor 5 for the displacement in the direction of the axis of rotation 14 is stationary and the motor 2 rotates the gantry, a circular trajectory is obtained for the radiation source and the detector unit 16 relative to the examination zone 13. Hereinafter only the

30   helical trajectory will be considered.

The measuring values acquired by the detector unit 16 are applied to an image processing computer 10 which is connected to the detector unit 16, for example, via a wireless data transmission (not shown). The image processing computer 10 reconstructs the absorption distribution in the examination zone 13 and reproduces it, for example, on a

monitor 11. The two motors 2 and 5, the image processing computer 10, the radiation source
S and the transfer of the measuring values from the detector unit 16 to the image processing
computer 10 are controlled by a control unit 7.

In other embodiments the acquired measuring values can first be applied to
5   one or more reconstruction computers for reconstruction, said computers applying the
reconstructed data to the image processing computer, for example, via an optical fiber cable.

Fig. 2 shows a flow chart illustrating a version of a measuring and
reconstruction method that can be executed by means of the computer tomograph shown in
Fig. 1.

10   After the initialization in the step 101, the gantry rotates at an angular speed
which is constant in the present embodiment. However, it may also vary, for example, in
dependence on the time or on the radiation source position. In the step 103 the examination
zone, or the object or the patient table, is displaced parallel to the axis of rotation and the
radiation of the radiation source S is switched on, so that the detector unit 16 can detect the
15   radiation from a plurality of angular positions.

In order to understand the next steps, reference is made to the following
equation from "Analysis of an Exact Inversion Algorithm for Spiral Cone-Beam CT",
Physics Medicine and Biology, Vol. 47, pp. 2583-2597:

20
$$f(\mathbf{x}) = -\frac{1}{2\pi^2} \int_{I_{Pl(x)}} ds \frac{1}{|\mathbf{x} - \mathbf{y}(s)|} \int_{-\pi}^{\pi} \frac{d\gamma}{d\sin\gamma} \frac{\partial}{\partial q} D_f(\mathbf{y}(q), \Theta(s, \mathbf{x}, \gamma))|_{q=s} . \tag{1}$$

This equation describes an exact reconstruction of the absorption by back
projection of the measuring values. Therein, f(x) denotes the spatial absorption distribution in
the examination zone in the location x and $I_{Pl}(x)$ describes the part of the helix which is
25   enclosed by a PI line 31.

The PI line 31 of an object point 35 in the location x in the examination zone
and $I_{PI(x)}$ are shown in Fig. 3 and Fig. 4 and will be described in detail hereinafter. The
radiation source moves relative to the examination zone around an object point 35 on a
helical path 17. The PI line 31 then is the line which intersects the helix in two locations and
30   the object point 35, the helical segment $I_{PI}(x)$ enclosed by the line then covering an angle
smaller than 2π.

Furthermore, in the equation (1) the reference s is the angular position of the
radiation source S on the helix related to an arbitrary but fixed reference angular position and
the reference y(s) is the position of the radiation source in three-dimensional space.

6

The measuring value $D_f(y,\Theta)$ can be described by the following line integral

$$D_f(\mathbf{y}(q), \Theta) = \int_0^\infty dl \ f(\mathbf{y} + l\Theta).$$

(2)

The unity factor Θ therein indicates the direction of the ray associated with the measuring value.

5   In the step 105 in conformity with equation (1) the measuring values are partially derived according to q, that is, in conformity with the angular position of the radiation source in the location $q=s$. In this respect it is to be noted that only y is dependent on q and not Θ, so that measuring values of parallel rays have to be taken into account for the derivation. Parallel rays have the same cone angle, the cone angle of a ray being the angle

10   enclosed by the projection of the ray in the xz plane of the co-ordinate system shown in Fig. 1 relative to the ray which extends through the axis of rotation and perpendicular thereto. As is shown in Fig. 5, rays having the same cone angle are incident on the same detector row in the case of a focus-centered detector, so that for the partial derivation measuring values of the same row but from different projections are taken into account. The derivation can then take

15   place, for example, by means of the finite difference method.

The unity factor Θ is dependent on the K angle γ which can be described by means of the so-called K planes 51. The K planes 51 will be described in detail hereinafter.

In order to determine a K plane 51 a function

$$s_1(s,s_2) = \begin{cases} \dfrac{ms_2 + (n-m)s}{n}, & s \le s_2 < 2 + 2\pi \\[2mm] \dfrac{ms + (n-m)s_2}{n}, & s > s_2 > s - 2\pi \end{cases}$$

20   (3)

is introduced, which function is dependent on non-negative, integer values n and m, where n>m. In this embodiment n=2 and m=1. However, other values n, m may also be chosen. The equation (1) would nevertheless remain exact, and only the position of the K planes 51 would change. Furthermore, the vector function

25

$$
\mathbf{u}(s,s_2) = \begin{cases} \dfrac{[\mathbf{y}(s_1(s,s_2)) - \mathbf{y}(s)] \times [\mathbf{y}(s_2) - \mathbf{y}(s)]}{\left|[\mathbf{y}(s_1(s,s_2)) - \mathbf{y}(s)] \times [\mathbf{y}(s_2) - \mathbf{y}(s)]\right|} \cdot \mathrm{sgn}(s_2 - s), & 0 < |s_2 - s| < 2\pi \\[2ex] \dfrac{\dot{\mathbf{y}}(s) \times \ddot{\mathbf{y}}(s)}{|\dot{\mathbf{y}}(s) \times \ddot{\mathbf{y}}(s)|}, & s_2 = s \end{cases}
$$

(4)

and the unity vector

$$
\beta(s,\mathbf{x}) = \frac{\mathbf{x} - \mathbf{y}(s)}{|\mathbf{x} - \mathbf{y}(s)|}
$$

(5)

are defined. The vector β then points from the radiation source position $y(s)$ to the position x.
In order to determine the K plane, a value $s_2 \, \varepsilon \, I_{PI(x)}$ is chosen so that $y(s)$, $y(s_1(s, s_2))$ $y(s_2)$ and
x are situated in one plane. This plane is referred to as the K plane 51 and the line of
intersection between the K plane 51 and the detector surface is referred to as the K line 53.
Fig. 6 shows a fan-like part of a K plane. The edges of the fan meet at the location of the
radiation source. This definition of the K plane 51 is equivalent to solution of the equation

$$(\mathbf{x} - \mathbf{y}(s)) \cdot \mathbf{u}(s, s_2) = 0, \quad s_2 \in I_{PI(x)} \tag{6}$$

according to s_2. Thus, u is thus the normal vector of the K plane 51. In order to determine the
vector function $\Theta(s,x,y)$ the vector

$$\mathbf{e}(s,\mathbf{x}) = \cos\gamma \cdot \beta(s,\mathbf{x}) + \sin\gamma \cdot \mathbf{e}(s,\mathbf{x}) \tag{7}$$

is defined. Using the definition for β and e, the vector function $\Theta(s,x,y)$ can be expressed as
follows:

$$\Theta(s,\mathbf{x},\gamma) = \cos\gamma \cdot \beta(s,\mathbf{x}) + \sin\gamma \cdot \mathbf{e}(s,\mathbf{x}) \tag{8}$$

Because both vectors β and e are oriented perpendicularly to u, the K angle γ indicates the
direction of the vector Θ and hence the direction of a ray within a K plane.

        The K planes and K lines are described in detail in E1 which is explicitly
referred to herein.

In the step 107 the measuring values derived along K lines are multiplied by a weighting factor, corresponding to the inverse sine of the K angle γ, and integrated in conformity with the equation (1). To this end, for each location x in the examination zone and for each projection angle there is determined a K line; as described above, a value $s_2 \varepsilon I_{PI(x)}$ is

5   then chosen to be such that y(s), $y(s_1(s,s_2))$, $y(s_2)$ and x are situated in one plane, that is, the K plane. The K line is then determined as the line of intersection between the K plane and the detector surface. The multiplications by the weighting factor and the integrations can be performed, for example, by means of Fourier filtering.

The derived and integrated measuring values can be represented by the

10   following equation:

$$p(y(s), \phi(s, x)) = \int_{-\Pi}^{\Pi} \frac{d\gamma}{\sin\gamma} \frac{\partial}{\partial q} D_f(y(q), \Theta(s,x,\gamma))|_{q=s}. \qquad (9)$$

Therein, $p(y(s),\Phi(s,x))$ denote the derived and integrated measuring values and $\Phi(s,x)$ is a

15   unity factor which points from the radiation source position y(s) in the direction of the location x in the examination zone.

The missing integration step in the equation (1) or the back projection of the measuring values can now be described by the following equation:

$$f(x) = -\frac{1}{2\pi^2} \int_{I_{PI(x)}} ds \frac{1}{|x - y(s)|} p(y(s), \Phi(s,x)). \qquad (10)$$

20   In conformity with this equation each measuring value must be multiplied by the factor 1/|x-y(s)| in order to reconstruct the spatial absorption distribution in the examination zone. This factor is dependent on the location x, so that it must be calculated anew for each combination of the radiation source position y(s) and the location x. Moreover, the integration over s takes place along the segment of the helix $I_{PI(x)}$. Thus, the integration

25   interval is dependent on the location x in which the absorption is to be determined, so that the integration interval must be determined for each location x. Because the integration in conformity with the equation (10) would require a large amount of calculation work for these reasons, the integration variable s is replaced by the projection angle φ hereinafter. The projection angle φ is then the angle enclosed by the PI line of the object point x projected in a

30   plane perpendicular to the axis of rotation (referred to hereinafter as the xy plane) and the

projection on the xy plane of the ray which passes through the location x while emanating
from the radiation source.

The integration in the equation (10) is carried out along the helical segment
$I_{PI(x)}$. This segment is enclosed by the PI line, so that an integration must be performed for
each measuring value from 0 to π after substitution of the integration variables. Thus, the
integration interval is the same for each location x in the examination zone.

The relationship between the integration variables ds and dφ can be derived
from Fig. 7. This Figure shows a projection of a helix 17, the object point 35 in the location x
and the radiation source positions y(s) and y(s+ds) on the xy plane. The following equation
results from Fig. 7:

$$d\varphi = \frac{\mid P_{xy}(\mathbf{y}) \mid ds\cos\varepsilon}{\mid P_{xy}(\mathbf{x-y}) \mid} = \frac{Rds\cos\varepsilon}{\sqrt{(x_x - y_x)^2 + (x_y - y_y)^2}}.$$

(11)

Therein, P_{xy} denotes the projection operator for the projection of a vector in the xy plane and
R denotes the radius of the helix 17. The fan angle ε is the angle enclosed by the normal from
the radiation source position to the axis of rotation and the projection of the ray which
emanates from the radiation source position and passes through the location x on the xy
plane. The indices x and y describe x and y components of a vector. The components relate to
the cartesian co-ordinate system shown in Fig. 1.

Thus, the following equation can be derived for the cone angle λ:

$$\cos\lambda = \frac{\sqrt{(x_x - y_x)^2 + (x_y - y_y)^2}}{\mid x - y \mid}.$$

(12)

The equations (10), (11) and (12) yield

$$f(\mathbf{x}) = -\frac{1}{2\pi^2} \int_0^\pi d\varphi \frac{\cos\lambda}{R\cos\varepsilon} p(y(s(\varphi)), \Phi(s(\varphi), x)).$$

(13)

In conformity with this equation, in the step 109 the measuring values are
multiplied by a first weighting factor, corresponding to the cosine of the cone angle λ, and by

a second weighting factor which corresponds to the reciprocal value of the cosine of the fan angle ε. Moreover, the measuring values can be multiplied by the inverse radius R. Because the radius is constant during the acquisition, the latter multiplication can also be performed after the back projection.

5   For small angles λ and ε, the multiplication by the weighting factors $\cos(\lambda)$ and $1/\cos(\varepsilon)$ can be ignored, because the cosine of these angles is then approximately one.

            The weighting factors in the step 109 are dependent on the cone angle λ and the fan angle ε. The weighting factors are thus the same for all locations x in the examination zone which are traversed by the same ray, meaning that for these locations the weighting

10   factors have to be calculated only once. In comparison with the known weighting by the weighting factor $1/|x-y(s)|$ of the equation (1), the foregoing leads to a substantial reduction of the required amount of calculation work.

            Prior to the back projection rebinning of the measuring values can be performed in the step 111. As a result of the rebinning operation the measuring values are

15   resorted and re-interpolated as if they had been measured by means of a different radiation source (an elongate radiation source which is arranged on a part of a helix and is capable of emitting each time mutually parallel fan beams) and by means of a different detector (a flat, rectangular "virtual" detector containing the axis of rotation 14).

            This will be described in detail with reference to Fig. 8. Therein, the reference

20   numeral 17 denotes the helical trajectory wherefrom the radiation source irradiates the examination zone. The reference numeral 43 denotes a fan-shaped radiation beam which emanates from the radiation source position S_0 and whose rays propagate in a plane containing the axis of rotation 14. The conical radiation beam emitted by the radiation source in the position S_0 can be assumed to consist of a plurality of flat fan beams which are situated

25   in planes which are parallel to the axis of rotation 14 and intersect in the radiation source position S_0. Fig. 8 shows only a single one of these fan beams, that is, the fan beam 43.

            Moreover, Fig. 6 shows further fan beams 41, 42 and 44, 45 which extend parallel to the fan beam 43 and are situated in planes which are parallel to one another and to the axis of rotation 14. The associated radiation source positions S_{-2}, S_{-1} and S_1, S_2 are

30   occupied by the radiation source S before and after having reached the radiation source position S_0, respectively. All rays in the fan beams 41 to 45 have the same projection angle.

            The fan beams 41 to 45 define a radiation beam 70 having a tent-like shape. For each group of fan beams there is defined a rectangular, virtual detector 160 which is situated in a plane which contains the axis of rotation 14 and is oriented perpendicularly to

the parallel fan beams of a group. The corner points of the virtual detector 160 constitute the
puncture points of the rays, incident on the oppositely situated helical segment from the outer
radiation source positions, through this plane. For the fan beam 70 in Fig. 8 these are the
points of intersection of the fan beams 41 and 45 with the helix. On the rectangular detector
160 detector there are defined elements which are arranged in a cartesian fashion, that is, in
rows and columns, on which the measuring values are re-interpolated.

The measuring values obtained after the rebinning are subsequently used for
the reconstruction of the absorption distribution in the examination zone by a back projection
which is in this embodiment in conformity with the equation (13).

In the step 113 a voxel $V(x,y,z)$ is determined within a selectable (x,y,z) zone
(field of view or FOV). Subsequently, in the step 115 a projection angle is selected within the
range $[\varphi_0, \varphi_0 + \pi]$ is selected, where φ_0 is the angle at which the voxel $V(x,y,z)$ enters the
radiation beam. In the step 117 it is checked whether a ray of the projection extends through
the center of the voxel $V(x,y,z)$. If no ray of the projection passes through the center of the
voxel, the associated value must be determined by interpolation of the measuring values of
neighboring rays. The measuring value that can be associated with the ray passing through
the voxel, or the measuring value obtained by interpolation, is accumulated on the voxel
$V(x,y,z)$ in the step 119. In the step 121 it is checked whether all projections with the
projection angles φ_0 to $\varphi 0+\pi$ have been taken into account. If this is not the case, the flow
chart branches to the step 115. Otherwise, it is checked in the step 123 whether all voxels
$V(x,y,z)$ in the FOV have been dealt with. If this is not the case, the procedure continues with
the step 113. However, when all voxels $V(x,y,z)$ in the FOV have been dealt with, the
absorption has been determined in the entire FOV and the reconstruction method is
terminated (step 125).

LIST OF REFERENCES:

1	gantry
2, 5	motor
3	collimator arrangement
4	conical radiation beam
7	control unit
10	image processing computer
11	monitor
13	examination zone
14	axis of rotation
16	detector unit
17	helix
31	PI line
35	object point
41 ... 45	fan-shaped radiation beam
51	K plane
53	K line
160	virtual detector
S	radiation source
$S_{-2} ... S_2$	radiation source positions.